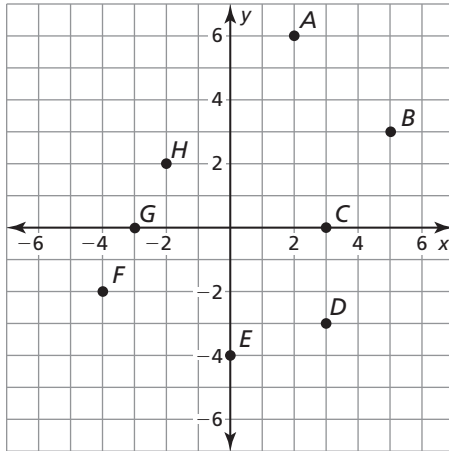


**Chapter
4****Maintaining Mathematical Proficiency**

Use the graph to answer the question.



1. What ordered pair corresponds to point A ?
2. What ordered pair corresponds to point H ?
3. What ordered pair corresponds to point E ?
4. Which point is located in Quadrant III?
5. Which point is located in Quadrant IV?
6. Which point is located on the negative x -axis?

Solve the equation for y .

7. $x - y = -12$

8. $8x + 4y = 16$

9. $3x - 5y + 15 = 0$

10. $0 = 3y - 6x + 12$

11. $y - 2 = 3x + 4y$

12. $6y + 3 - 2x = x$

13. Rectangle $ABCD$ has vertices $A(4, -2)$, $B(4, 5)$, and $C(7, 5)$. What are the coordinates of vertex D ?

4.1

Writing Equations in Slope-Intercept Form

For use with Exploration 4.1

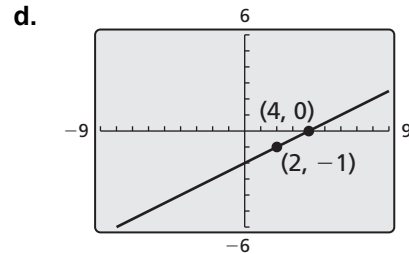
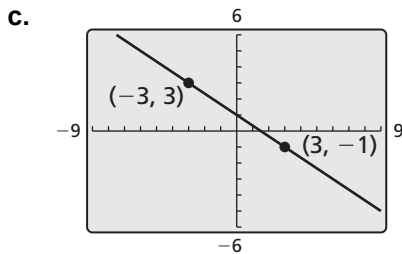
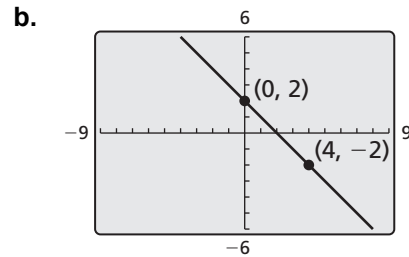
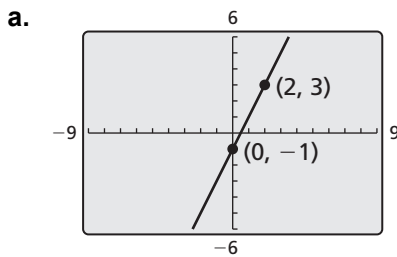
Essential Question Given the graph of a linear function, how can you write an equation of the line?

1 EXPLORATION: Writing Equations in Slope-Intercept Form

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

- Find the slope and y -intercept of each line.
- Write an equation of each line in slope-intercept form.
- Use a graphing calculator to verify your equation.

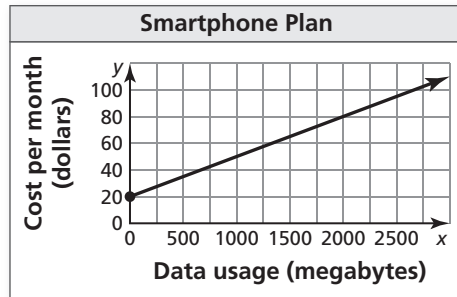


4.1 Writing Equations in Slope-Intercept Form (continued)

2 EXPLORATION: Mathematical Modeling

Work with a partner. The graph shows the cost of a smartphone plan.

- a. What is the y -intercept of the line?
Interpret the y -intercept in the context of the problem.

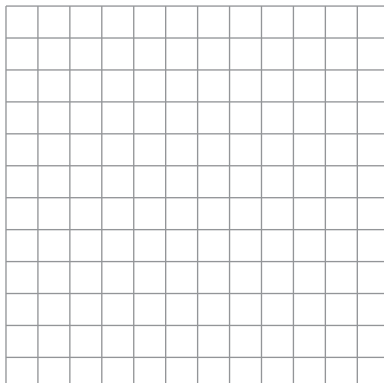


- b. Approximate the slope of the line.
Interpret the slope in the context of the problem.

- c. Write an equation that represents the cost as a function of data usage.

Communicate Your Answer

- 3. Given the graph of a linear function, how can you write an equation of the line?
- 4. Give an example of a graph of a linear function that is different from those above. Then use the graph to write an equation of the line.



4.1

Notetaking with Vocabulary

For use after Lesson 4.1

In your own words, write the meaning of each vocabulary term.

linear model

Notes:

4.1 Notetaking with Vocabulary (continued)

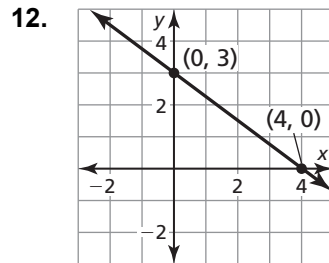
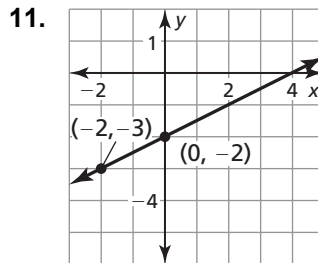
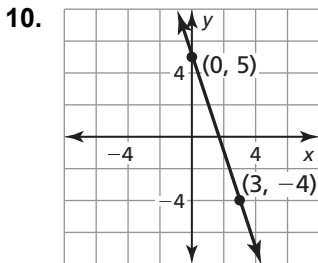
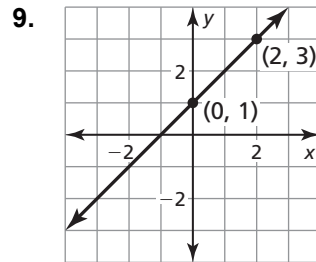
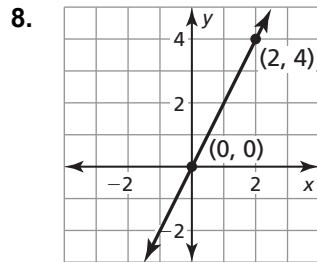
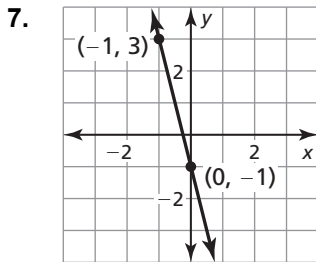
Extra Practice

In Exercises 1–6, write an equation of the line with the given slope and y-intercept.

- | | | |
|-------------------------------|----------------------------------|----------------------------------|
| 1. slope: 0
y-intercept: 9 | 2. slope: -1
y-intercept: 0 | 3. slope: 2
y-intercept: -3 |
|-------------------------------|----------------------------------|----------------------------------|

- | | | |
|----------------------------------|----------------------------------|---|
| 4. slope: -3
y-intercept: 7 | 5. slope: 4
y-intercept: -2 | 6. slope: $\frac{1}{3}$
y-intercept: 2 |
|----------------------------------|----------------------------------|---|

In Exercises 7–12, write an equation of the line in slope-intercept form.



4.1 Notetaking with Vocabulary (continued)

In Exercises 13–18, write an equation of the line that passes through the given points.

13. $(0, -4), (8, 4)$

14. $(2, 1), (0, -7)$

15. $(0, 2), (4, 3)$

16. $(0, -5), (-4, -1)$

17. $(8, 0), (0, 8)$

18. $(0, 3), (2, -5)$

In Exercises 19–24, write a linear function f with the given values.

19. $f(0) = -5, f(4) = -3$

20. $f(-5) = 5, f(0) = 10$

21. $f(0) = 5, f(9) = -4$

22. $f(0) = 10, f(7) = -4$

23. $f(-2) = -2, f(0) = 2$

24. $f(0) = 16, f(2) = 8$

25. An electrician charges an initial fee of \$50 and \$190 after 4 hours of work.

a. Write a linear model that represents the total cost as a function of the number of hours worked.

b. How much does the electrician charge per hour?

4.2**Writing Equations in Point-Slope Form**

For use with Exploration 4.2

Essential Question How can you write an equation of a line when you are given the slope and a point on the line?

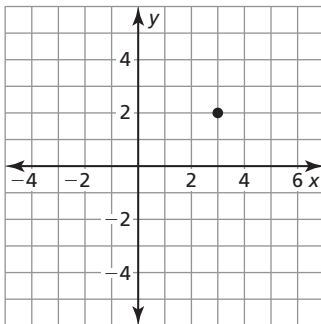
1 EXPLORATION: Writing Equations of Lines

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

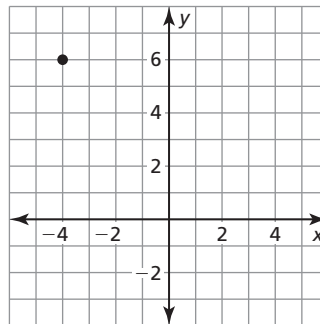
Work with a partner.

- Sketch the line that has the given slope and passes through the given point.
- Find the y -intercept of the line.
- Write an equation of the line.

a. $m = \frac{1}{2}$

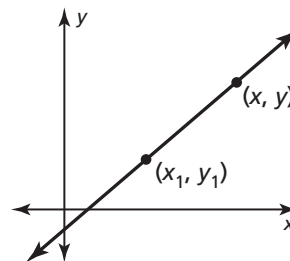


b. $m = -2$

**2 EXPLORATION: Writing a Formula**

Work with a partner.

The point (x_1, y_1) is a given point on a nonvertical line. The point (x, y) is any other point on the line. Write an equation that represents the slope m of the line. Then rewrite this equation by multiplying each side by the difference of the x -coordinates to obtain the **point-slope form** of a linear equation.



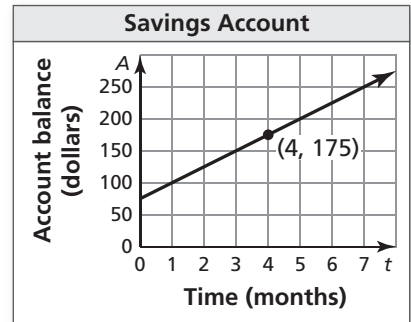
4.2 Writing Equations in Point-Slope Form (continued)**3** **EXPLORATION:** Writing an Equation

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

For four months, you have saved \$25 per month.
You now have \$175 in your savings account.

- a. Use your result from Exploration 2 to write an equation that represents the balance A after t months.



- b. Use a graphing calculator to verify your equation.

Communicate Your Answer

4. How can you write an equation of a line when you are given the slope and a point on the line?
5. Give an example of how to write an equation of a line when you are given the slope and a point on the line. Your example should be different from those above.

4.2 Notetaking with Vocabulary (continued)

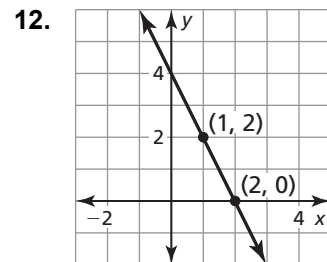
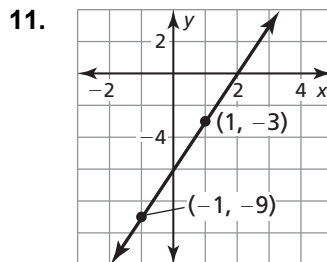
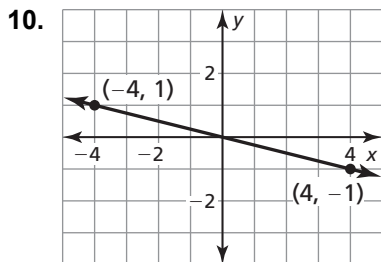
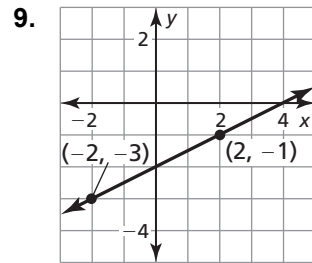
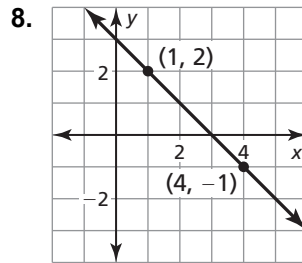
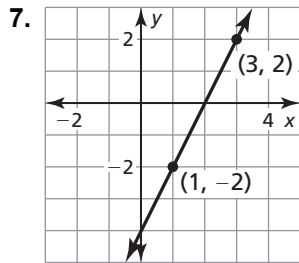
Extra Practice

In Exercises 1–6, write an equation in point-slope form of the line that passes through the given point and has the given slope.

1. $(-2, 1); m = -3$ 2. $(3, 5); m = 2$ 3. $(-1, -2); m = -1$

4. $(5, 0); m = \frac{4}{3}$ 5. $(0, 4); m = 7$ 6. $(1, 2); m = -\frac{1}{2}$

In Exercises 7–12, write an equation in slope-intercept form of the line shown.



4.2 Notetaking with Vocabulary (continued)

In Exercises 13–18, write a linear function f with the given values.

13. $f(-3) = -1, f(-2) = 4$ 14. $f(-2) = 1, f(1) = 7$ 15. $f(-1) = 2, f(3) = 3$

16. $f(0) = -2, f(4) = -1$ 17. $f(1) = 0, f(0) = 8$ 18. $f(3) = 5, f(2) = 6$

In Exercises 19 and 20, tell whether the data in the table can be modeled by a linear equation. Explain. If possible, write a linear equation that represents y as a function of x .

19.

x	-3	-1	0	1	3
y	-110	-60	-35	-10	40

20.

x	-3	-1	0	1	3
y	-98	18	8	62	142

21. Your friend is driving at a constant speed of 60 miles per hour. After driving 3 hours, his odometer reads 265 miles. Write a linear function D that represents the miles driven after h hours. What does the odometer read after 7 hours of continuous driving?

4.3**Writing Equations of Parallel and Perpendicular Lines**

For use with Exploration 4.3

Essential Question How can you recognize lines that are parallel or perpendicular?

1 EXPLORATION: Recognizing Parallel Lines

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Write each linear equation in slope-intercept form. Then use a graphing calculator to graph the three equations in the same square viewing window. (The graph of the first equation is shown.) Which two lines appear parallel? How can you tell?

a. $3x + 4y = 6$

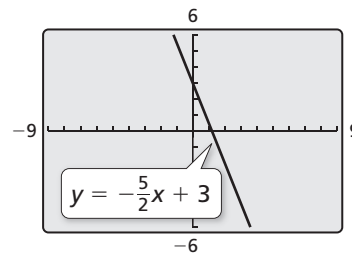
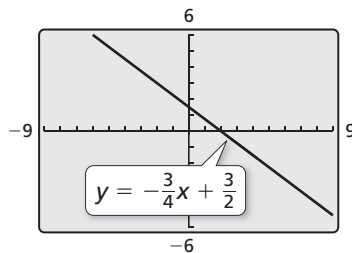
b. $5x + 2y = 6$

$3x + 4y = 12$

$2x + y = 3$

$4x + 3y = 12$

$2.5x + y = 5$



4.3 Writing Equations of Parallel and Perpendicular Lines (continued)**2 EXPLORATION: Recognizing Perpendicular Lines**

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Write each linear equation in slope-intercept form. Then use a graphing calculator to graph the three equations in the same square viewing window. (The graph of the first equation is shown.) Which two lines appear perpendicular? How can you tell?

a. $3x + 4y = 6$

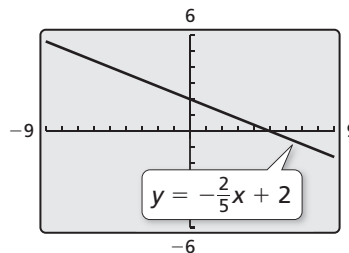
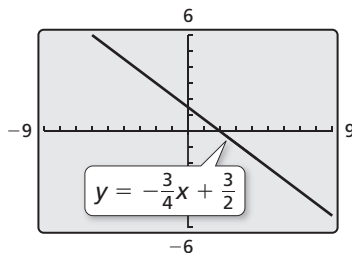
b. $2x + 5y = 10$

$3x - 4y = 12$

$-2x + y = 3$

$4x - 3y = 12$

$2.5x - y = 5$

**Communicate Your Answer**

- How can you recognize lines that are parallel or perpendicular?
- Compare the slopes of the lines in Exploration 1. How can you use slope to determine whether two lines are parallel? Explain your reasoning.
- Compare the slopes of the lines in Exploration 2. How can you use slope to determine whether two lines are perpendicular? Explain your reasoning.

4.3**Notetaking with Vocabulary**

For use after Lesson 4.3

In your own words, write the meaning of each vocabulary term.

parallel lines

perpendicular lines

Core Concepts**Parallel Lines and Slopes**

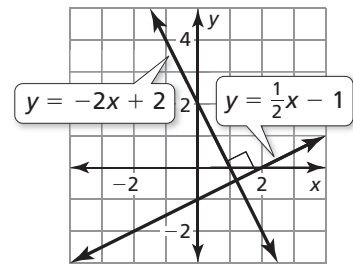
Two lines in the same plane that never intersect are **parallel lines**. Two distinct nonvertical lines are parallel if and only if they have the same slope.

All vertical lines are parallel.

Notes:**Perpendicular Lines and Slopes**

Two lines in the same plane that intersect to form right angles are **perpendicular lines**. Nonvertical lines are perpendicular if and only if their slopes are negative reciprocals.

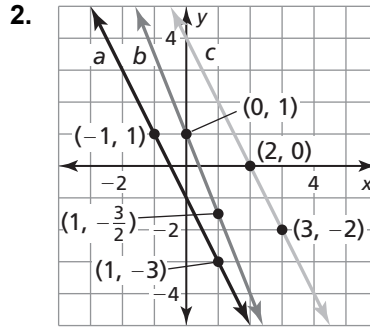
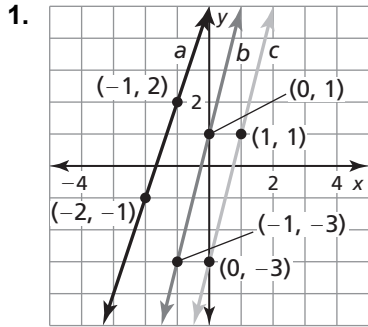
Vertical lines are perpendicular to horizontal lines.

Notes:

4.3 Notetaking with Vocabulary (continued)

Extra Practice

In Exercises 1–6, determine which of the lines, if any, are parallel. Explain.



3. Line a passes through $(-4, -1)$ and $(2, 2)$.
 Line b passes through $(-5, -3)$ and $(5, 1)$.
 Line c passes through $(-2, -3)$ and $(2, -1)$.

4. Line a passes through $(-2, 5)$ and $(2, 1)$.
 Line b passes through $(-4, 3)$ and $(3, 4)$.
 Line c passes through $(-3, 4)$ and $(2, -6)$.

5. Line a : $4x = -3y + 9$
 Line b : $8y = -6x + 16$
 Line c : $4y = -3x + 9$

6. Line a : $5y - x = 4$
 Line b : $5y = x + 7$
 Line c : $5y - 2x = 5$

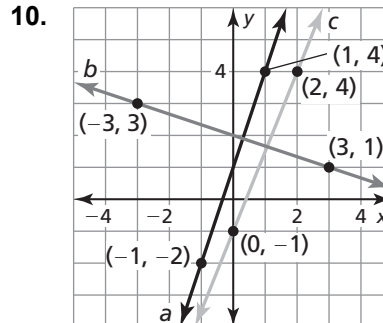
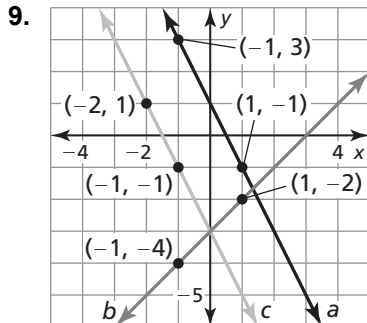
In Exercises 7 and 8, write an equation of the line that passes through the given point and is parallel to the given line.

7. $(3, -1)$; $y = \frac{1}{3}x - 3$

8. $(1, -2)$; $y = -2x + 1$

4.3 Notetaking with Vocabulary (continued)

In Exercises 9–14, determine which of the lines, if any, are parallel or perpendicular. Explain.



11. Line a passes through $(-2, 4)$ and $(1, 1)$.
Line b passes through $(2, 1)$ and $(4, 4)$.
Line c passes through $(1, -2)$ and $(-1, 4)$.
12. Line a passes through $(-2, -4)$ and $(-1, -1)$.
Line b passes through $(-1, -4)$ and $(1, 2)$.
Line c passes through $(2, 3)$ and $(4, 2)$.

13. Line a : $y = \frac{3}{4}x + 1$
Line b : $-3y = 4x - 3$
Line c : $4y = -3x + 3$

14. Line a : $5y - 2x = 1$
Line b : $y = \frac{5}{2}x - 1$
Line c : $y = \frac{2}{5}x + 3$

In Exercises 15 and 16, write an equation of the line that passes through the given point and is perpendicular to the given line.

15. $(-2, 2)$; $y = \frac{2}{3}x + 2$
16. $(3, 1)$; $2y = 4x - 3$

4.4**Scatter Plots and Lines of Fit**

For use with Exploration 4.4

Essential Question How can you use a scatter plot and a line of fit to make conclusions about data?

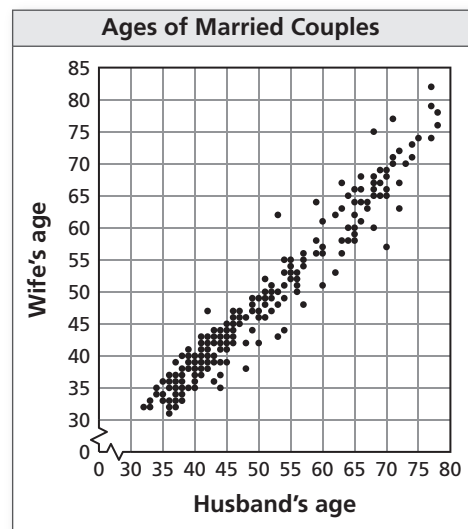
A **scatter plot** is a graph that shows the relationship between two data sets. The two data sets are graphed as ordered pairs in a coordinate plane.

1 EXPLORATION: Finding a Line of Fit

Go to BigIdeasMath.com for an interactive tool to investigate this exploration.

Work with a partner. A survey was taken of 179 married couples. Each person was asked his or her age. The scatter plot shows the results.

- a. Draw a line that approximates the data. Write an equation of the line. Explain the method you used.



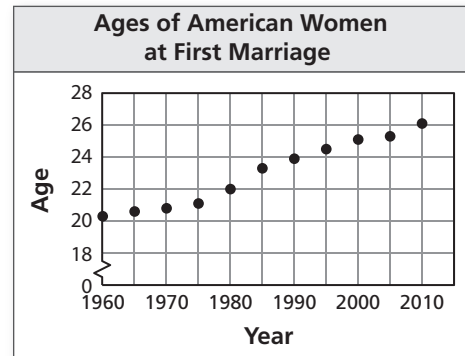
- b. What conclusions can you make from the equation you wrote? Explain your reasoning.

4.4 Scatter Plots and Lines of Fit (continued)**2 EXPLORATION:** Finding a Line of Fit

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. The scatter plot shows the median ages of American women at their first marriage for selected years from 1960 through 2010.

- Draw a line that approximates the data. Write an equation of the line. Let x represent the number of years since 1960. Explain the method you used.
- What conclusions can you make from the equation you wrote?
- Use your equation to predict the median age of American women at their first marriage in the year 2020.

**Communicate Your Answer**

- How can you use a scatter plot and a line of fit to make conclusions about data?
- Use the Internet or some other reference to find a scatter plot of real-life data that is different from those given above. Then draw a line that approximates the data and write an equation of the line. Explain the method you used.

4.4

Notetaking with Vocabulary

For use after Lesson 4.4

In your own words, write the meaning of each vocabulary term.

scatter plot

correlation

line of fit

Core Concepts

Scatter Plot

A **scatter plot** is a graph that shows the relationship between two data sets. The two data sets are graphed as ordered pairs in a coordinate plane. Scatter plots can show trends in the data.

Notes:

4.4 Notetaking with Vocabulary (continued)**Using a Line of Fit to Model Data**

Step 1 Make a scatter plot of the data.

Step 2 Decide whether the data can be modeled by a line.

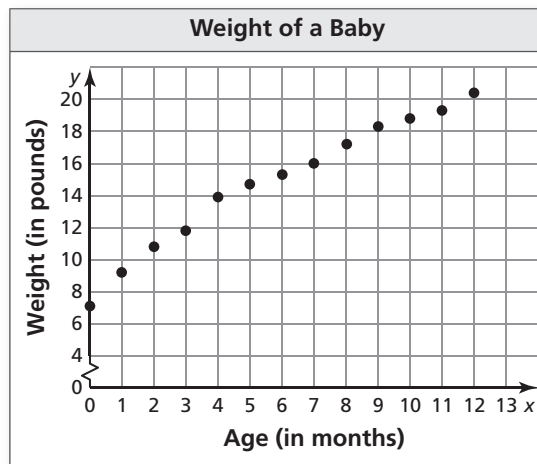
Step 3 Draw a line that appears to fit the data closely. There should be approximately as many points above the line as below it.

Step 4 Write an equation using two points on the line. The points do not have to represent actual data pairs, but they must lie on the line of fit.

Notes:

Extra Practice

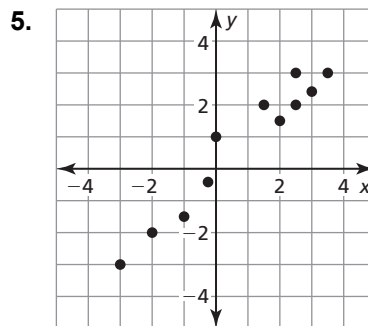
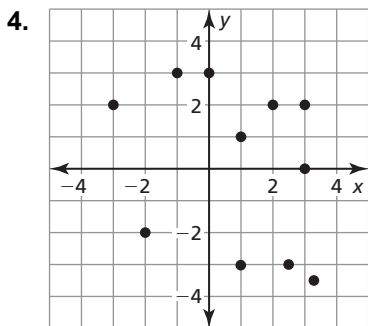
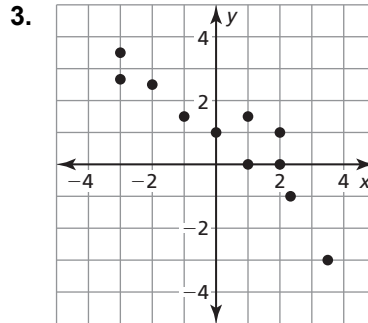
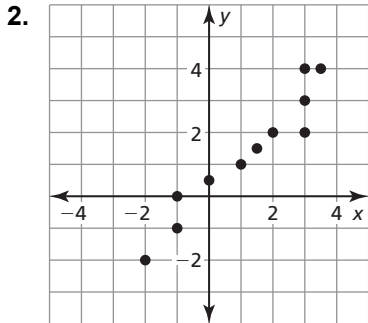
1. The scatter plot shows the weights (in pounds) of a baby over time.



- a. What is the weight of the baby when the baby is four months old?
- b. What is the age of the baby when the baby weighs 17.2 pounds?
- c. What tends to happen to weight of the baby as the age increases?

4.4 Notetaking with Vocabulary (continued)

In Exercises 2–5, tell whether x and y show a *positive*, a *negative*, or *no* correlation.



6. The table shows the depth y (in centimeters) of water filling a bathtub after x minutes.

Time (minutes), x	0	2	4	6	8	10	12
Depth (centimeters), y	6	8	11	14	17	20	24

a. Write an equation that models the depth of the water as a function of time.

b. Interpret the slope and y -intercept of the line of fit.

4.5

Analyzing Lines of Fit

For use with Exploration 4.5

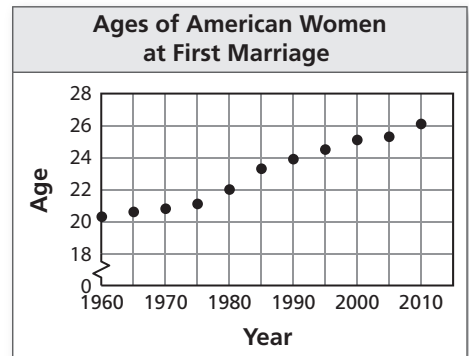
Essential Question How can you *analytically* find a line of best fit for a scatter plot?

1 EXPLORATION: Finding a Line of Best Fit

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner.

The scatter plot shows the median ages of American women at their first marriage for selected years from 1960 through 2010. In Exploration 2 in Section 4.4, you approximated a line of fit graphically. To find the line of *best* fit, you can use a computer, spreadsheet, or graphing calculator that has a *linear regression* feature.



- a. The data from the scatter plot is shown in the table. Note that 0, 5, 10, and so on represent the numbers of years since 1960. What does the ordered pair (25, 23.3) represent?

L1	L2	L3
0	20.3	
5	20.6	
10	20.8	
15	21.1	
20	22	
25	23.3	
30	23.9	
35	24.5	
40	25.1	
45	25.3	
50	26.1	

L1(55)=		

- b. Use the *linear regression* feature to find an equation of the line of best fit. You should obtain results such as those shown below.

```
LinReg
y=ax+b
a=.1261818182
b=19.84545455
r2=.9738676804
r=.986847344
```

- c. Write an equation of the line of best fit. Compare your result with the equation you obtained in Exploration 2 in Section 4.4.

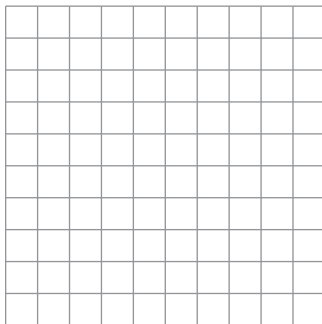
4.5 Analyzing Lines of Fit (continued)**Communicate Your Answer**

2. How can you *analytically* find a line of best fit for a scatter plot?

3. The data set relates the number of chirps per second for striped ground crickets and the outside temperature in degrees Fahrenheit. Make a scatter plot of the data. Then find an equation of the line of best fit. Use your result to estimate the outside temperature when there are 19 chirps per second.

Chirps per second	20.0	16.0	19.8	18.4	17.1
Temperature (°F)	88.6	71.6	93.3	84.3	80.6

Chirps per second	14.7	15.4	16.2	15.0	14.4
Temperature (°F)	69.7	69.4	83.3	79.6	76.3



4.5

Notetaking with Vocabulary

For use after Lesson 4.5

In your own words, write the meaning of each vocabulary term.

residual

linear regression

line of best fit

correlation coefficient

interpolation

extrapolation

causation

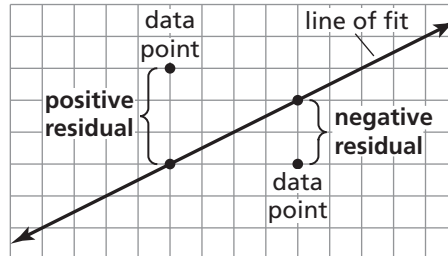
Notes:

4.5 Notetaking with Vocabulary (continued)

Core Concepts

Residuals

A **residual** is the difference of the y -value of a data point and the corresponding y -value found using the line of fit. A residual can be positive, negative, or zero.



A scatter plot of the residuals shows how well a model fits a data set. If the model is a good fit, then the absolute values of the residuals are relatively small, and the residual points will be more or less evenly dispersed about the horizontal axis. If the model is not a good fit, then the residual points will form some type of pattern that suggests the data are not linear. Wildly scattered residual points suggest that the data might have no correlation.

Notes:

Extra Practice

In Exercises 1 and 2, use residuals to determine whether the model is a good fit for the data in the table. Explain.

1. $y = -3x + 2$

x	-4	-3	-2	-1	0	1	2	3	4
y	13	11	8	6	3	0	-4	-8	-10

4.5 Notetaking with Vocabulary (continued)

2. $y = -0.5x + 1$

x	0	1	2	3	4	5	6	7	8
y	2	0	-3	-5	-7	-6	-4	-3	-1

3. The table shows the numbers y of visitors to a particular beach and the average daily temperatures x .

- a. Use a graphing calculator to find an equation of the line of best fit. Then plot the data and graph the equation in the same viewing window.

Average Daily Temperature (°F)	Number of Beach Visitors
80	100
82	150
83	145
85	190
86	215
88	263
89	300
90	350

- b. Identify and interpret the correlation coefficient.
- c. Interpret the slope and y -intercept of the line of best fit.

4.6

Arithmetic Sequences

For use with Exploration 4.6

Essential Question How can you use an arithmetic sequence to describe a pattern?

An **arithmetic sequence** is an ordered list of numbers in which the difference between each pair of consecutive **terms**, or numbers in the list, is the same.

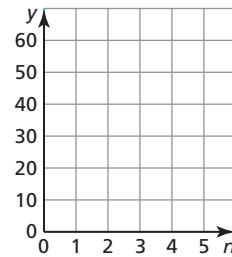
1 EXPLORATION: Describing a Pattern

Go to *BigIdeasMath.com* for an interactive tool to investigate this exploration.

Work with a partner. Use the figures to complete the table. Plot the points given by your completed table. Describe the pattern of the y -values.

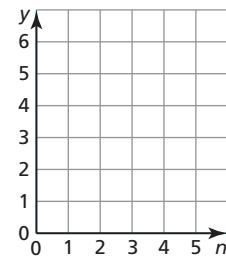
a. $n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$

Number of stars, n	1	2	3	4	5
Number of sides, y					



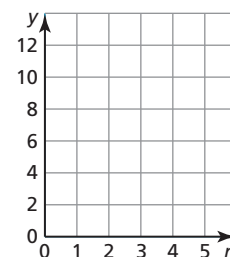
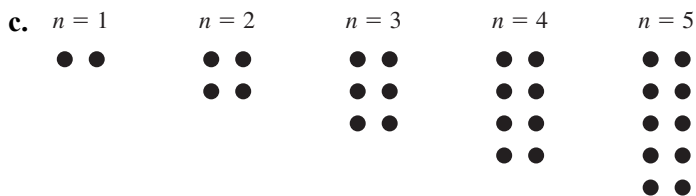
b. $n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$

n	1	2	3	4	5
Number of circles, y					



4.6 Arithmetic Sequences (continued)

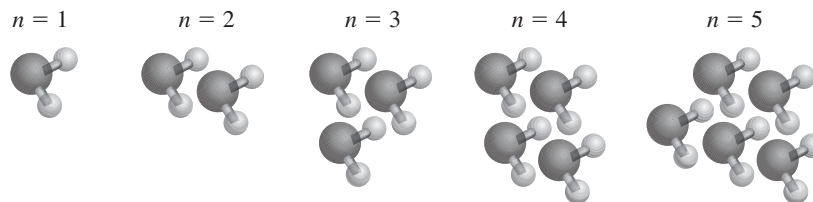
1 EXPLORATION: Describing a Pattern (continued)



Number of rows, n	1	2	3	4	5
Number of dots, y					

Communicate Your Answer

- How can you use an arithmetic sequence to describe a pattern? Give an example from real life.
- In chemistry, water is called H_2O because each molecule of water has two hydrogen atoms and one oxygen atom. Describe the pattern shown below. Use the pattern to determine the number of atoms in 23 molecules.



4.6**Notetaking with Vocabulary**

For use after Lesson 4.6

In your own words, write the meaning of each vocabulary term.

sequence

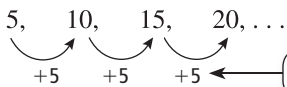
term

arithmetic sequence

common difference

Core Concepts**Arithmetic Sequence**

In an **arithmetic sequence**, the difference between each pair of consecutive terms is the same. This difference is called the **common difference**. Each term is found by adding the common difference to the previous term.



Terms of an arithmetic sequence

common difference

Notes:**Equation for an Arithmetic Sequence**

Let a_n be the n th term of an arithmetic sequence with first term a_1 and common difference d . The n th term is given by

$$a_n = a_1 + (n - 1)d.$$

Notes:

4.6 Notetaking with Vocabulary (continued)

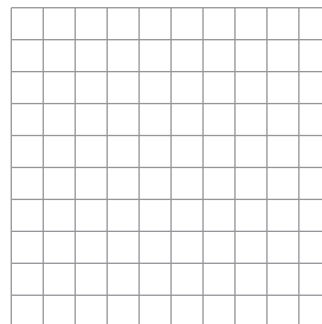
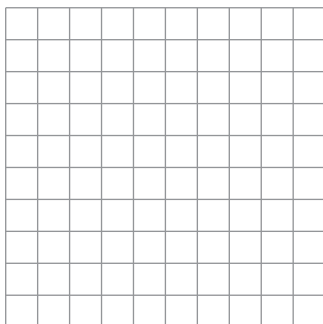
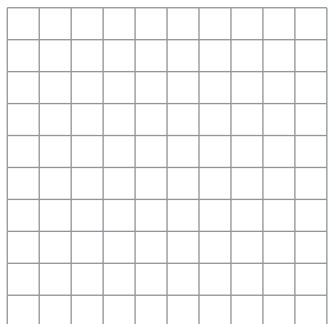
Extra Practice

In Exercises 1–6, write the next three terms of the arithmetic sequence.

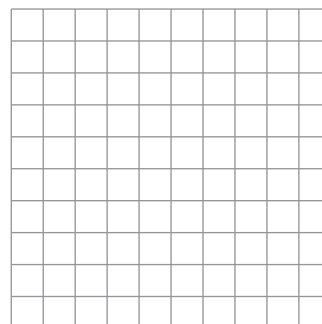
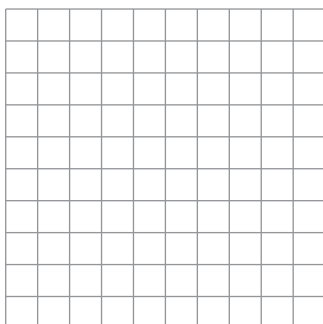
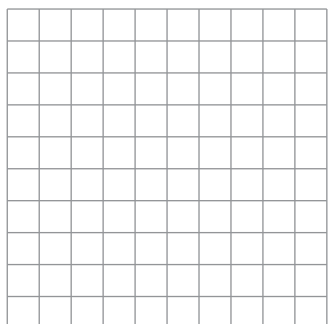
1. 1, 8, 15, 22, ... 2. 20, 14, 8, 2, ... 3. 12, 21, 30, 39, ...
4. 5, 12, 19, 26, ... 5. 3, 7, 11, 15, ... 6. 2, 14, 26, 38, ...

In Exercises 7–12, graph the arithmetic sequence.

7. 1, 3, 5, 7, ... 8. 9, 6, 3, 0, ... 9. $\frac{15}{2}, \frac{13}{2}, \frac{11}{2}, \frac{9}{2}, \dots$

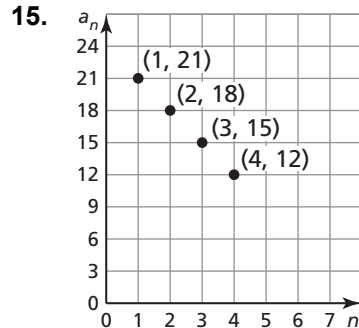
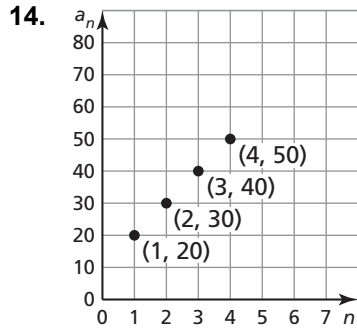
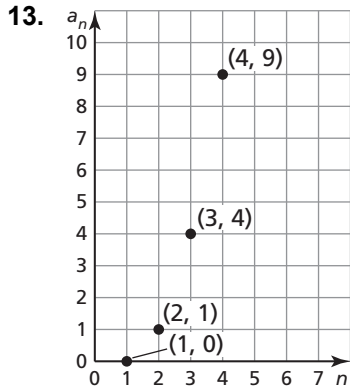


10. 1, 2.5, 4, 5.5, ... 11. 1, 4, 7, 10, ... 12. $\frac{1}{4}, \frac{5}{4}, \frac{9}{4}, \frac{13}{4}, \dots$



4.6 Notetaking with Vocabulary (continued)

In Exercises 13–15, determine whether the graph represents an arithmetic sequence. Explain.



In Exercises 16–21, write an equation for the n th term of the arithmetic sequence. Then find a_{10} .

16. $-5.4, -6.6, -7.8, -9.0, \dots$

17. $43, 38, 33, 28, \dots$

18. $6, 10, 14, 18, \dots$

19. $-11, -9, -7, -5, \dots$

20. $34, 37, 40, 43, \dots$

21. $\frac{9}{4}, \frac{7}{4}, \frac{5}{4}, \frac{3}{4}, \dots$

22. In an auditorium, the first row of seats has 30 seats. Each row behind the first row has 4 more seats than the row in front of it. How many seats are in the 25th row?